

# Foreign exchange and stock market: two related markets?

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## **Abstract**

This paper studies the relationship between the stock market and the exchange rate in several countries. The approach taken in the first part of this study is a linear VAR, to be compared in the following part to a MS-VAR. The data is also analyzed by Granger causality tests in both contexts and a thorough description of the empirical results obtained is shown. The research uncovers a spread (but not constant over time) causality from the exchange rate and American stock market to the local markets of the different nations studied. The non-linear, time varying approach allows several considerations on the dynamics of the relationship. The markets analyzed are the Japanese, the British and the German (pre-Euro) market against the US Dollar and the US stock market. The frequency of the data used is daily.

## Introduction

The search for a relationship between the exchange rates and the national stock markets has been popular for several decades. A wide array of branches of economics and business finance have dealt with it in the common goal of increasing their understanding of any or both the markets based on their dynamic interaction with other, seemingly similar in characteristics, markets. After all the stock and the foreign exchange market have often been described by similar adjectives and in a considerable amount of contexts the literature applies the same models in the explanation of both. Therefore the natural academic curiosity as well as financial research on this relationship.

However, in the past such a general topic has given few decisive hints on a single approach to take. The variety of the backgrounds of the researchers involved has provided a wide range of theories and approaches. To this day there is no single orthodox approach on the subject, and the existing literature is rich in enlightening contributions coming from several sources.

In the search for the right approach to take and for the originating phenomena that are at the basis of the relationship, it is interesting to verify the results of several simple empirical tests on the time series of a selection of developed markets. The questions asked in this first step should be: which market Granger causes which? Is there a long term relationship? Is the relationship strong in the short term? Is the relationship stable over time? Answers to this first set of queries will be at the base of the following analysis to be carried on. As a matter of fact often it is possible to verify if a relationship detected in a sample of data is to be confirmed by repeating the analysis on the same time series, in different periods. A key factor that has recently gained importance on foreign exchange orthodox literature has been the non-linearity of its relationship in several aspects. Since the time series is not linear with respect to itself [14], or with respect to the fundamentals of its economies [3], it might as well not be linear in its relationship with the stock market. On the other hand, just as this finding has opened several possibilities and new theories in the study of exchange rate series, it does so also in the analysis of stock market series.

The above developments show in a nutshell why it is very reasonable (to say the least) to conceive a non-linear, and possibly time-varying dynamics at the basis of the relationship in study. This would at once rationalize the very valid contributions to this subject in such different sciences and approaches as in the current literature. All of the sound economic developments in the different approaches

could indeed be true in the same time series, if considering varying regimes over time.

The last question left before continuing on to the description of the main existing theoretical backgrounds is what tools to use to verify such time-variation of the underlying dynamics. For this purpose there exist a set of models whose main quality is its flexibility that will be used in this paper. Markov Switching Models (MSM) allow for the data to verify the duration and definition of the specifications involved, and through a 2 step maximum likelihood estimation process characterizes each of the alternating regimes. This relatively new process has been promptly applied in several fields and topics. However very few are the papers that use this tool in the study of the subject at hand.

The remaining part of this essay will be structured as follows. Section 1 will provide an overview of the main approaches in which the current literature may be subdivided. This will be useful in the interpretation of the results found in the body of the paper. Section 2 will describe the data studied and the methodology used to analyze it. Reasons for the adoption of specific methodologies will be discussed and a critical review of some of the procedures will be provided. Section 3 will carry out the empirical analysis chosen in order to study the data long run behavior. This section will also describe and implement the MS analysis and verify its appropriateness in the context. Section 4 will provide an interpretation of the results obtained through the different analysis for all the markets studied. The results will be checked against other macroeconomic variables and connections will be traced. Finally section 5 will offer some critical analysis of the main findings and conclude with the room left for research.

## **1 Literature review and theoretical feedback**

The literature is rich in contributions to different theories or approaches, but most of them can be categorized in two main classes. Each of the classes has in turn several different models or analysis according to the specifics of the single case in itself. The first of the two classes shows how from the macro point of view it is possible to find logical and economical two-way feedback between the markets, that can therefore change as influenced by the stronger pressure from time to time. On the other hand it is also possible to show the micro point of view. In this class the decisions of the single companies and major institutions is kept in mind, and the practical influence of the exchange rate on the financial choices of the company is analyzed. Also in this case a two-way causality is possible, according to the

different circumstances that will arise.

From the economic point of view it is possible to trace a relationship between the stock market behavior and the exchange rate dynamics. In analyzing the flows in the economies it is possible from economic theory to state that exchange rate performance does affect the real economy through international competitiveness and therefore on the balance of trade. The logical link between this and the company cash flows is immediate, just as it is to the stock prices themselves. On the other hand economic theory also provides an inverted direction of causality, from the stock market to the exchange rate. This is because equities are by definition a very important fraction of wealth. Therefore swings in equity prices will affect the demand for money and finally the exchange rate determination.

This approach has been strengthened in 1980 by the contribution of Dornbush and Fisher [16] in which they set the tone for an economic point of view of the subject matter. Exchange rates are studied in the relationship to the current account and therefore a flow analysis is very important to their proposition and point. Several other articles have continued on this approach. However the mentioned paper remains at the base of this line of thinking.

The second class, more based on the micro approach and on stock measures, takes the company decisions in consideration. This is why the measure of company exposure plays a key role in this class of articles. Several are the examples of original contributions in this aspect. Branson [11] and Frankel [19] in 1983 show models of how this approach is developed within a company's financial decisions based on the supply and demand for stock and bonds. Franck and Young [18] show a remarkable analysis of the scenario and are the first to be interested in this relationship. The focus on multinational decision making process is of sure example to future researchers. However they find no relationship between the markets despite the study of six separate cases.

The work of Aggarwal published in 1981 [2] is a very important contribution in the literature, and analyzes the dollar and its stock prices, finding a positive short term dynamics. However since then other papers have been published finding differing results. Soenen and Hennigar [51] follow a slightly different empirical approach (more conservative on the measure of the dollar value) and find a negative correlation. Smith [49, 50] makes use of a model on which he bases his conclusions and papers. Following the Portfolio Balance Model, he finds that indeed this relationship is very important in the specification of any model that would explain the exchange rates. The paper by Granger, Huang and Yang [20] studies several Asian markets from a Granger Causality point of view and finds several

interactions, that are different depending on the markets studied.

A very interesting paper has been published in 2003 by Pavlova and Rigobon [42]. The authors use a two-country, two-good model to describe the behavior of the real exchange rate, the stock and the bond markets. They make predictions that are inspired by the concept that exchange rates behave by the same principles as the stock market, and should therefore be treated in a similar manner. Their predictions seem to be confirmed by the data and are therefore a welcome contribution to the literature.

Another analysis of particular interest has been carried out by Phylaktis and Ravazzolo [43] and it takes the macro approach to study the time series in their short and long term correlation. The analysis has focused on some of the Pacific Basin Countries and it has found several causality relationships, including the one of the US market on the markets of the economies analyzed. The relationship is studied both in its long and short term phenomena, and observes in particular the channels that are most successful in describing the dynamics.

Lastly, a paper that has introduced a new empirical tool in this analysis has been the work (published in 2006) of Priestley and Odegaard [44], where they study the exchange rate exposure of the different industries in different periods of time. The underlying concept remains that the market reacts differently in appreciating regimes than in the depreciating ones. Therefore it is possible and all the more useful to use Markov Switching Models in the analysis to verify how the variables change over time. This paper shows coherent conclusions that confirm the starting hypothesis of the adequacy of this scheme for the varying relationship of the exchange rate to the stock market.

## 2 Data and methodology

In order to cover a wide amount of cases, several markets have been selected. The United States dollar always acts as a counterpart to the exchange rates analyzed and the American market is taken as a comparing market for the stock prices. The three scenarios studied are against the British market (1991 through 2006), the Japanese market (1991 through 2006) and the pre-Euro German case (1991-1999). All the series are on a daily basis to provide a different analysis from most of the other papers in the literature. Using this shorter horizon it is our hope that it will be possible to identify trends and phenomena that are not detected when studying more spread series. However, although the daily base allows for a focus on short term analysis, this should not take emphasis away from uncovering longer term

dynamics, since the study will analyze a time-period of 15 years in all except the German case (where the period observed is of 7 years and 2 months).

Following the approach of Phylaktis and Ravazzolo [43] we perform a cointegration analysis to uncover any long term trend. Then we will proceed accordingly with either a VECM (in case the data is cointegrated) or with a VAR. The objective of this procedure is going to be the study of the coefficients of the system, and analysis of the influence that each series has on the other. The model can be written as follows:

$$S_t = \alpha_0 + \alpha_1 A_t^{US} - \alpha_2 A_t + \varepsilon_t \quad (1)$$

The variables have been considered in returns to focus on the change of the variables in relation to the changes in the other variables. Therefore the time series have been prepared to have  $S$  as the change of the exchange rate of the dollar over the foreign currency in logarithms.  $A^{US}$  and  $A$  are the returns on the prices of the stock indexes for the United States (SP500) and for the studied market. Finally  $\varepsilon$  is the time series of the disturbance term. All the data has been used in logarithms.<sup>1</sup> However it is important, before preparing the series in returns, to verify whether they are cointegrated, and in case they are, what are the cases in which the long term cointegrating vector is significant. This will be the base for the further analysis (for example it will give us the ground to decide for a VAR process on the returns or for a VECM). Once the relationship has been defined on the returns through the specification of the  $\alpha$  values, then it is interesting to apply a Granger causality test to understand the influence direction throughout the time series. After these tests have been carried out for the different markets on the whole sample available, it is possible to check whether the causality that may be found using this linear technique is subject to change depending on the sample period analyzed. To make sure that we have solid reasons for breaking the series in smaller sections, it is important to verify the characteristics of the relationship between the exchange rate and the stock market. For this purpose a new VAR analysis will be carried out that will be parallel to the first in structure but more flexible. The new added flexibility will be due to a Markov switching model that will define different coefficients for the VAR process according to the varying properties of the relationship over time. Several regimes will be estimated

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<sup>1</sup>As a consequence of this paragraph the exchange rate series used for this paper is calculated as:

$$S_t = (\ln(e_t) - \ln(e_{t-1}))/\ln(e_{t-1})$$

that will alternate on the original sample, and will allow the VAR specifications to be more precise in the short term. It will be possible to describe the different regimes according to the coefficients found, and understand the alternating real pressures on the markets. The procedure used to estimate these regimes is going to be based on the Expectation-Maximization procedure developed by Hamilton [22] and refined according to the VAR structure.<sup>2</sup>

This new procedure can be described formally by the following expression:

$$X_t = \alpha_0(r_t) + \sum_{i=1}^{k-1} \alpha_{i,j}(r_t) X_{t-i} + \varepsilon_t \quad (2)$$

where  $r_t$  is the regime specific variable, and causes the coefficients to be different depending on the value of  $s$  in every moment in time. In this case  $X_t$  is taken as an example of a variable including all the time series described above ( $S$ ,  $A$ , and  $A^{US}$ ). It is clear that in this specification of the model the constant will be regime specific as well as the coefficients of the VAR process. Furthermore the model is studied to allow heteroschedasticity through the different variance values allowed in each of the regimes.

The last issue that will be discussed in the methodology of this paper will be the procedure used to choose the value  $s$  of the number of regimes. This will be crucially important, given that depending on the choice of regimes, the non-linear models obtained will have different specifications. Finally, once the regimes have been described and studied it will be interesting to focus on the periods that make up the different states of nature and test the data for Granger causality. In this manner we can verify if by applying a non-linear procedure we can confirm or deepen the results obtained with the linear technique. Finally the results will be commented.

## 2.1 A note on MS model selection

In order to properly implement Markov Switching modelling, it is important to take an objective standpoint in the selection of the number of regimes that will alternate in every market. This problem has been faced in different manners by several economists and scientists in general. However, despite the ongoing debate on the better procedures for model selection, only a minority of the existing papers on MSM use any of the procedures put forth. The result of this chain of events

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<sup>2</sup>To this purpose a program will be used through the GAUSS software that is based on the original version by Bellone [9]

is that although a number of different solutions has been presented, few of these have indeed been implemented in the literature. To this day there is one orthodox procedure that, due to its history and power in a range of different models has been put forth, and several other suggestions described in lone standing papers. The goal of this section is to describe the main procedure, discuss it, and conclude with an acceptance or a refusal of that procedure based on logical implications. In case of a refusal, another procedure will be suggested to solve the problem at hand.

The most widely used procedure is the Likelihood ratio test. This test has gained importance in the literature due to the strong application of its underlying concept and to the impeccable logic in selecting the best among two different models at hand. However in the selection of the number of regimes in a Markov Switching model the underlying concept is slightly different than in the other array of models. The difference is made by the time factor: once the number of regimes has been chosen there is a univocal relationship to the specification of all the VAR coefficients in the model. Naturally, a higher number of regimes will allow the model to make a closer portrait of the data. The likelihood will be increasing directly with the number  $s$  of regimes. The data analyzed in this paper makes no exception and in all three markets the Log Likelihood value does structurally grow if we consider 3 regimes rather than 2 (just like it does with increasing the number of regimes above 3). So the question that the test should solve would be solely if the increase of  $LL$  (the Log Likelihood) is small enough to allow the acceptance of the Null hypothesis or if  $H_0$  cannot be accepted given the result. The definition of the Null hypothesis is also to regulate, since the nature of the test itself gives it a conservative approach towards the Null, therefore making it biased towards this decision.

In its general form, the LR statistic is defined as follows:

$$LR = 2[LL(\hat{\vartheta}_{N+1}) - LL(\hat{\vartheta}_N)] \quad (3)$$

where  $LL(\hat{\vartheta}_{N+1})$  and  $LL(\hat{\vartheta}_N)$  are the Log Likelihood functions of the models that have as parameter specifications the sample sets  $\hat{\vartheta}_{N+1}$  and  $\hat{\vartheta}_N$ .

However MSM are so that not all parameters are identified under both hypothesis, so the regularity conditions are violated, and the asymptotic chi-square distribution typical of the LRT can no longer be proven. In a nutshell, this means that in order to answer the question described above, another procedure needs to be described that will amend the problem of the LRT to make it applicable to MSM. Thanks to Hansen [23, 24, 25] a new procedure has been defined to re-



calculate all the distributions by which to judge the size of the LR according to the different model configurations in analysis. However, this procedure has been applied in a very limited number of cases in literature, and only partially due to its computational complexity [12]. For example Hansen defines clearly the case of  $N = 1$  versus a model that is multi-regime. His goal is to analyze whether a series is time-dependent in a first place or not. However he does not dwell into the case of an  $N$  value bigger than 1. Furthermore the test has been studied by Cheung Erlandsson [12]. They describe Hansen's asymptotic derivation and instead adopt the Monte Carlo approach to define the sensitive distributions and conclude with an array of concepts describing the issues that the LRT has in dealing with MSM. After applying the test in both directions ( $N$  vs.  $N + 1$  as well as  $N + 1$  vs.  $N$ ), they show that the results will be different based on the direction, as discussed above. Also, their Monte Carlo procedures demonstrate how the conclusions based on the test performance are based on the frequency of the data as well as on the size of the sample.

Overall it is clear that the question remains on the size of the LR for this to be accepted as statistically insignificant or else for  $H_0$  to be rejected, but no agreed way has been found yet to measure the significance of the ratio, especially given that:

- the likelihood value keeps increasing with the number of regimes specified;
- the asymptotic distribution typical of the LRT is not longer valid, and therefore a new ad-hoc distribution needs to be agreed upon (there is no yet an orthodox, widely accepted solution for the procedure to use in case the regularity conditions are not respected).

As discussed above, in the literature there exist other procedures that have been thought out to solve the problem instead of the LRT, but none has yet (to my knowledge) taken a lead. For more information on other procedures it is possible to refer to a paper by Psaradakis and Spagnolo [45]. They list and compare a number of different procedures with consistent results.

Indeed it is interesting to notice how lately a new branch of procedures is being developed to study the results of MSM and compare them, and determine the right number of regimes for every market based on those results. This approach assumes, first of all, that there exists a "right" number of regimes, and that the outcome of the estimation will give the necessary information to verify whether one value of  $N$  is more correct than another. A good example of this procedure can be found

in Layton and Smith [35], where a 2 and 3 regime scenario are compared and one of the two is chosen based on solid MS logic considerations. This branch of models is spreading as a solution to the problem and as an alternative to model selection procedures that seem not to be apt to suit the nature of time-varying, regime switching models. The details of this experimental procedure will be given in the course of the paper, as in the next few sections the three markets are analyzed, and the number of regimes will be singularly chosen for each scenario, based on the results obtained from the elaboration of the data. For the moment it will suffice to state the underlying concept of the procedure and the questions it asks in order to select the best model:

- the EM procedure set up to calculate the coefficients given the data has a strong logic that should be respected to make decisions concerning the specification of the model;
- once the data is studied with this procedure information is given that will not necessarily arise from other procedures, and this information is mainly provided to the researcher through the matrix of transition probabilities and through the vector of final probabilities;
- in analyzing the information provided through the elaboration of different models the same information will be interpreted in different ways, and not all these ways will be the best interpretation of the data;
- the procedure used to find the best specification should therefore use the results provided by the different models and single out the best logical interpretation of the information provided by the data.

## 3 Empirical results

### 3.1 Cointegration analysis

The series in levels will now be analyzed for cointegration. The presence of a cointegrating vector would mark the existence of a long term relationship between the asset prices of the two countries and the exchange rate. As a consequence the second part of the paper will show the model structured following these results. In the presence of such a long term relationship the non linear analysis performed will concentrate on the short term dynamics changes. Specifically, the long term relationship would be kept constant across the regimes, while only the coefficients

Nation	$\ln(e)$	$\ln(P)$
US	X	0.56
JAP	0.13	0.12
UK	0.44	0.57
GE	0.38	0.96

Table 1: Unit Root results: Probabilities that the  $H_0$  of “Unit root” is true.

Market	Trace	ME
US-JAP	0.3622	0.5406
US-UK	0.2435	0.3251
US-GE	0.0933	0.2250

Table 2: Cointegration results: Probabilities that the  $H_0$  of “No Cointegrating vector” is true.

of the VECM would be adjusted depending on the regime. This concept is important since economic theory assumes the cointegrating relationship of the data to be stable across time, given its long term nature.

Before applying the tests it is useful to verify how many and which series have unit roots. Table 1 illustrates the results of the tests performed. In the table  $\ln(e)$  and  $\ln(P)$  stand for the logs of the series in levels. The probabilities found show that all series used in this study have a unit root, therefore the cointegration analysis will provide useful information in the interpretation of the dynamics within the markets.

Table 2 shows how according to the Trace test and the Maximum Eigenvalue test no market has cointegrated series in levels. As a matter of fact all of the probabilities shown in the table are well above the conventional boundary of 5% and therefore leave no doubt on the procedure to follow as well as on the kind of dynamics to be uncovered. Given the lack of a long term linear relationship found in all the markets, the second part of this paper, in which non-linear procedures are used, gains a greater importance from the point of view of a long run interpretation of the data.

### 3.2 Linear VAR and overall Granger Causality

The cointegration analysis offered very important information in the understanding of the data in study. Given the results obtained a VAR process will be appropriate to estimate the influence that the different variables have on each other. The applied model in all the markets will be:

$$\begin{bmatrix} S_t \\ A_t^{US} \\ A_t \end{bmatrix} = \begin{bmatrix} \alpha_{10} \\ \alpha_{20} \\ \alpha_{30} \end{bmatrix} + \begin{bmatrix} \alpha_{11}(L) & \alpha_{12}(L) & \alpha_{13}(L) \\ \alpha_{21}(L) & \alpha_{22}(L) & \alpha_{23}(L) \\ \alpha_{31}(L) & \alpha_{32}(L) & \alpha_{33}(L) \end{bmatrix} \begin{bmatrix} S_{t-1} \\ A_{t-1}^{US} \\ A_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^S \\ \varepsilon_t^{A^{US}} \\ \varepsilon_t^A \end{bmatrix} \quad (4)$$

Equation 3 shows how the different markets can be tested for the influences that the variables have on each other. The degree of the AR process will depend for every market on the results of the Akaike, Schwarz and Hannan-Quinn information criteria. Therefore every market will have an independently chosen AR degree, while the structure of the model will remain similar.

In Table 3 concerning the specification for Japan it is possible to see how the stock market is influenced by the exchange rate as well as by the American stock fluctuations. This can be explained by the fact that stock markets in developed countries are often closely tied, and the exchange rate is a key factor in the transmission of shocks between the two economies. In the same table it can be argued that the US market is indeed a possible proxy for the world market, hypothesis upheld by the non-significance of the Japanese market in the American stock equation.

Table 4 shows the situation on the UK market. Similar trends to the previous market can be noticed. A very peculiar point is how the US stock price has a very strong statistical influence on the local market, as testified by the high t-values for all the lags of the  $A^{US}$  variable in the  $A$  equation. Another important aspect is how the British stock market shows also significance for the exchange rate with the dollar. The exchange rate equation on the other hand has a low but significative influence by the local as well as by the US stock market. Furthermore it is interesting to remark how the exchange rate seems to follow an AR process, since the influence of  $S(-1)$  is significative (t-value of 2.14).

Table 5 depicts the scenario in the German economy before the introduction of the Euro. Statistical significance considerations suggest a similar interpretation to the UK market. However it is important to remember how the sample period of this market ends about 8 years before the others. The American market heavily influenced the local Stock prices in that period. It is possible to wonder if the

	S	A	$A^{US}$
S(-1)	-0.016660 [-1.03900]	0.034853 [ 1.96805]	0.004536 [ 0.28713]
S(-2)	0.005174 [ 0.32258]	0.022664 [ 1.27938]	0.024675 [ 1.56131]
A(-1)	-0.011131 [-0.76336]	0.058958 [ 3.66079]	-0.019369 [-1.34810]
A(-2)	-0.010595 [-0.76901]	-0.018488 [-1.21501]	0.000977 [ 0.07198]
$A^{US}(-1)$	-0.006966 [-0.42575]	0.377096 [ 20.8672]	-0.012310 [-0.76356]
$A^{US}(-2)$	-0.023783 [-1.37950]	0.019702 [ 1.03471]	-0.013379 [-0.78756]
C	-2.64E-06 [-0.11297]	-2.25E-05 [-0.87316]	5.12E-05 [ 2.22392]

Table 3: US-Japan market: linear VAR specification

increased cohesion of the EU market since then might have lowered the influence shown by this specification. In any case the local stock market shows very high t-values for most coefficients. This is indeed characterized by changes in the exchange rate as well as by the American returns. The exchange rate variable is sensitive to changes in the American stock returns, while the US stock market shows no significance (confirming once again the proxy hypothesis).

As a conclusion for the linear analysis it is interesting and important to Granger test the variables for which variables are caused by which. Given the structure of the model and its properties it is preferable to use the multivariate version of the test. Therefore every variable will be singularly tested for the joint causality of the other two in its equation.

Table 6 shows the results of the Granger multivariate causality test on the Japanese market. The table shows the probabilities that the dependent variables are not caused by the other variables in the VAR. Therefore a rejection of the Hypothesis with probabilities close to 0 would imply a direction of causality from the two independent variables to the dependent one. The results for these tests appear to be fairly homogeneous both in the probability values and the consequences. Indeed, as shown in Table 6, the exchange rate appears to be independent of any

	S	A	$A^{US}$
S(-1)	0.034647 [ 2.13829]	-0.000172 [-0.15684]	0.001191 [ 0.57525]
S(-2)	0.010284 [ 0.63505]	-0.001718 [-1.56921]	-0.001770 [-0.85550]
S(-3)	-0.031067 [-1.91928]	-0.001771 [-1.61771]	0.001493 [ 0.72182]
S(-4)	0.034996 [ 2.16293]	-0.003657 [-3.34299]	0.002379 [ 1.15031]
A(-1)	-0.072673 [-0.28635]	-0.139870 [-8.15081]	0.051022 [ 1.57312]
A(-2)	-0.354964 [-1.40452]	-0.064046 [-3.74793]	-0.004694 [-0.14533]
A(-3)	-0.318572 [-1.26094]	-0.180963 [-10.5934]	-0.031586 [-0.97829]
A(-4)	-0.572882 [-2.35902]	-0.016020 [-0.97563]	0.071500 [ 2.30384]
$A^{US}(-1)$	-0.245394 [-1.84321]	0.178003 [ 19.7739]	-0.022971 [-1.35011]
$A^{US}(-2)$	0.282216 [ 2.00860]	0.025642 [ 2.69908]	-0.026859 [-1.49580]
$A^{US}(-3)$	0.087571 [ 0.62317]	0.039185 [ 4.12408]	-0.024671 [-1.37375]
$A^{US}(-4)$	0.123516 [ 0.88469]	0.023833 [ 2.52462]	-0.020622 [-1.15580]
C	0.000115 [ 0.63828]	3.06E-05 [ 2.50378]	5.11E-05 [ 2.21461]

Table 4: US-UK market: linear VAR specification

	S	A	$A^{US}$
S(-1)	0.002570 [ 0.10926]	0.015616 [ 7.78167]	0.001016 [ 0.51668]
S(-2)	0.015970 [ 0.66949]	0.004109 [ 2.01852]	0.001959 [ 0.98252]
A(-1)	-0.136600 [-0.49181]	-0.071593 [-3.02081]	0.007524 [ 0.32412]
A(-2)	-0.274481 [-1.15991]	-0.016647 [-0.82443]	-0.014576 [-0.73702]
$A^{US}(-1)$	0.666208 [ 2.35953]	0.600277 [ 24.9154]	-0.004219 [-0.17880]
$A^{US}(-2)$	-0.119058 [-0.36403]	-0.100159 [-3.58894]	-0.011482 [-0.42006]
C	9.93E-05 [ 0.28607]	2.65E-05 [ 0.89483]	9.69E-05 [ 3.34103]

Table 5: US-GE market: linear VAR specification

Markets	Dependent variables		
	S	A	$A^{US}$
JAP-US	0.3148	0.0000	0.3744
UK-US	0.0605	0.0000	0.2099
GER-US	0.1091	0.0000	0.7438

Table 6: Causality test on different subperiods. Probabilities that  $H_0$  is true.  $H_0$ : dependent variable not caused by the others

causality, while the local market is always caused by the other two variables. The results are very definite (probabilities of 0.0000) and equal for all the markets studied<sup>3</sup>. Having discussed the results it is possible to conclude that the direction of causality is certainly from the exchange rate and US stock market to the local market rather than the opposite. On the other hand the only case where the hypothesis of causality in this inverse direction is not definitely rejected is the British market, where the probability of multivariate independence is of 0.0605.

The above results show compact considerations that are confirmed in all the markets studied. The Granger causality test performances being similar are also an indication that the model used is theoretically solid and not subject to peculiar circumstances. However the same structure of the model that allows for these results is also not flexible to short term changes of the dynamics towards different directions. While the procedure is solid, it only provides a synthesis of the data over time.

### 3.3 MS-VAR modelling of the stock market-exchange rate relationship

In this paragraph the model will be kept with the same structure, but it will be hypothesized that several regimes or states of nature alternate over time in the description of the relationship in study. Every market will therefore be interpreted by a set of two or more specifications of the VAR.

#### 3.3.1 The Japanese market

The Japanese market has been analyzed using the MS model illustrated above. Indeed two distinct elaborations have been carried (for 2 regimes as well as for 3 regimes). In order to choose what exact specification to adopt for this market both of these possibilities are shown. In the case of 2 regimes the Markovian transitional probability matrix will be as follows:

$$P^{JAP}[i, j] = \begin{bmatrix} 0.932 & 0.155 \\ 0.068 & 0.845 \end{bmatrix} \quad (5)$$

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<sup>3</sup>Despite the multivariate nature of the tests it is also possible, in a nutshell and as further information provided, to add that in all but the Japanese case the exchange rate is a key factor in the causing of the local stock market variable. In the Japanese case the results are not definite, and leave space to interpretation (probability of  $H_0$  of the exchange rates causing the local market changes of 0.0661).



The Markovian transition probabilities are the probabilities that given regime  $i$  in period 0, the next period the system will be in regime  $j$ . Therefore, given that at any point in time the system will be in one of the two regimes, the sum of every column is always equal to 1. Furthermore, on the diagonal of the matrix there are the probabilities that the regime will be persistent and stay in period 1 in the same regime as it was in period 0. Given the nature of the MSM usually there will be high probabilities on the diagonal, signifying that the regimes are persistent and do not switch every single period. This property allows the use of this set of models to increase predictability due to the determination of separate, but persistent, regimes.

Another very important result of the estimation is going to be the absolute probability of being in one of the two regimes at a certain point in time. This is given for the Japanese MS(2)-VAR(2) case by the following vector:

$$P^{JAP} = \begin{bmatrix} 0.694 \\ 0.306 \end{bmatrix} \quad (6)$$

From the above vector we learn that in over a third of the cases the first regime is present instead of the second regime, present in only a minority of the periods in the sample. In the structure of the model this can be verified by observing the matrix of transition probabilities, which shows the first regime to be more persistent than the second one. So whenever the first regime is verified in period  $t - 1$  it is very likely for this same regime to be realized in period  $t$  rather than for the system to switch to regime 2. The smoothed probabilities for this model are shown in Figure 1.

At this point it is interesting to verify whether adding one more regime is going to add any more information to the set already collected after a first analysis with 2 regimes. The matrix of transition probabilities is shown below:

$$P^{JAP}[i, j] = \begin{bmatrix} 0.932 & 0.016 & 0.287 \\ 0.014 & 0.966 & 0.082 \\ 0.053 & 0.018 & 0.631 \end{bmatrix} \quad (7)$$

The probabilities listed above show that of the 3 regimes, one of them has a lower duration, since it only repeats itself period after period in 63% of the cases. The other two regimes, however, are very persistent, and support the hypothesis that this approach can give a valid support to the traditional VAR processes. However in order to decide which model (MS(2) or MS(3)) is more appropriate for this scenario, it is important to study the dynamics of the data. Observing

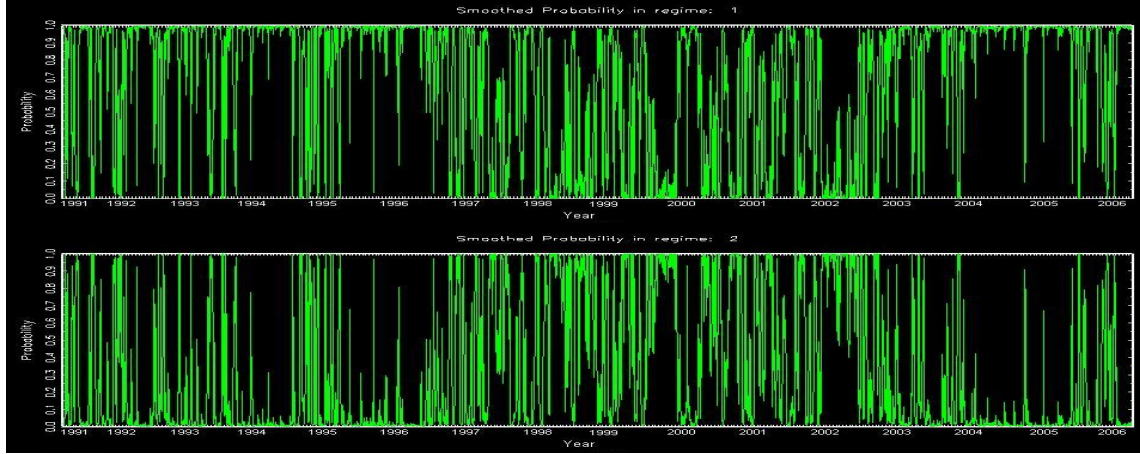


Figure 1: JAP-US: MS(2)-VAR(2), Smoothed probabilities of being in the two regimes defined at every period in time

Equation 6 it is possible to notice how if in period 0 the system is in regime 1, it is very likely to stay in regime 1 also in period 1. Furthermore if the system switches, it is over 3 times more likely to switch to regime 3 rather than to regime 2. So, following the dynamics, if it switches to regime 3, it stays there up to when it switches. At this point it is more likely to switch back to regime 1 rather than 2. This indicated a mild cycle 1-3 regime, letting regime 2 out. It is important to notice how, despite the described cycle, this should be referred to as mild, because there is still a very relevant probability (over 1/4) that the cycle will be broken. Having noticed the cycle and the strength of it, it is interesting to observe the ergodic (final or unconditional) probability vector for the 3 regime case:

$$P^{JAP} = \begin{bmatrix} 0.485 \\ 0.424 \\ 0.090 \end{bmatrix} \quad (8)$$

The third regime is shown to be present in about 1 in 10 periods, while regime 1 on itself is present in almost half the periods. Figure 2 illustrates how this is divided in the time series.

Looking at Figure 2 the concept expressed above becomes clear, as the first two regimes appear to have probabilities very close to 1 in specific points in time for longer sequences of periods. However the cycle appears clearly in the beginning of the second half of the time-series, where regimes 1 and 3 alternate and the

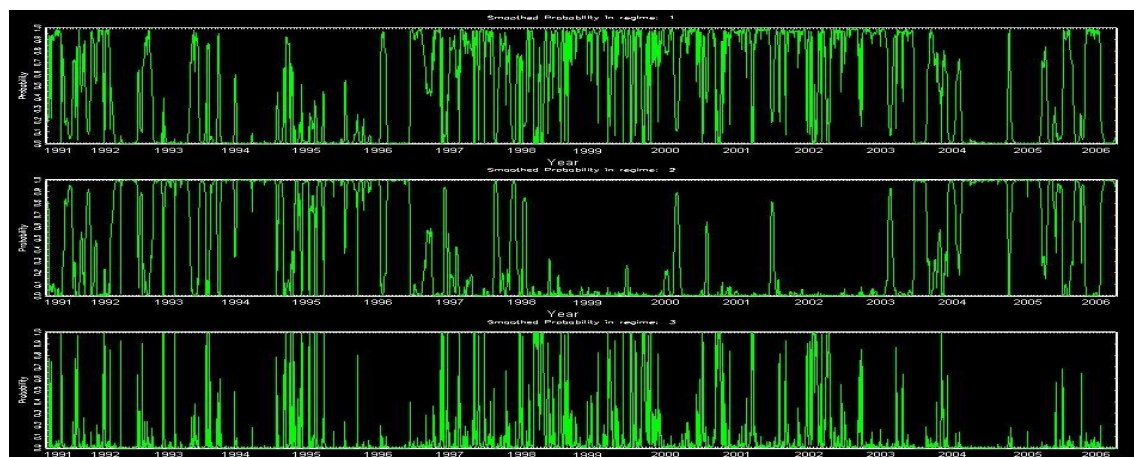


Figure 2: JAP-US: Smoothed probabilities of being in the three regimes defined at every period in time

probability of regime 2 is close to 0 for a fairly long stretch of time. Yet, that is the only clear episode where this phenomenon is shown.

It is important now to go back to the data for the MS(2) performance. If that specification of the model is more apt to describe this market versus the 3-regime specification, there should be clear signs that the information given in the MS(3) is well summarized in the MS(2) picture. Explicitly, the MS(3) data should not add any relevant information to that provided earlier. Given the strong cycle identified in the latter specification, if these two regimes are to be grouped in 1 single, more persistent regime, the probability matrixes should clearly show it. It could be expected that one of the two regimes could indeed be invariant while the other be the simple sum of probabilities of the two<sup>4</sup>. If the two regimes do not summarize the information in such a way, there is a good possibility that the system cannot be summarized by two regimes at all, and the 3-regime scenario does indeed add information that would be lost in any simpler model (just like it happened with the linear version of the model described previously).

In this case it is clear (from the Figures shown as well) that the two regime version of the model does not have any regime in common with the three regime and, conversely, no two regimes from the 3-regime version, make up any of the

<sup>4</sup>This phenomenon indeed happens when describing an artificially originated time series constructed from 2 regimes with anything else than a two regime explication. Every regime is divided into two or more regimes, according to the number of regimes specified time by time.

	S	A	$A^{US}$
C (Reg.1)	-0.0174	-0.0176	0.0223
(Reg.2)	-0.0344	0.0067	0.0763
(Reg.3)	0.2022	-0.1159	-0.1116
S(-1) (Reg.1)	-0.0326	0.0076	0.0118
(Reg.2)	-0.0593	-0.0992	0.0086
(Reg.3)	0.0566	-0.0775	-0.0495
S(-2) (Reg.1)	0.0021	0.0332	-0.0384
(Reg.2)	0.0333	0.0511	-0.0039
(Reg.3)	0.0051	0.1269	0.0225
A(-1) (Reg.1)	0.0330	0.4256	-0.0245
(Reg.2)	0.0476	0.3153	0.0065
(Reg.3)	-0.0693	0.3498	-0.0036
A(-2) (Reg.1)	0.0365	-0.0390	-0.0423
(Reg.2)	-0.0730	-0.0499	-0.0550
(Reg.3)	0.0329	0.0217	0.0421
$A^{US}(-1)$ (Reg.1)	0.0106	-0.0299	0.0064
(Reg.2)	0.0102	0.0118	-0.0127
(Reg.3)	0.0072	-0.0130	-0.0221
$A^{US}(-2)$ (Reg.1)	0.0140	0.0320	0.0013
(Reg.2)	0.0477	0.0555	-0.0327
(Reg.3)	0.0328	-0.0217	-0.0608

Table 7: JAP-US market: MS(3)-VAR(2) specification

two regimes shown in the simpler specification. It is at this point safe to say that the Japanese market cannot be correctly described by a MS(2) model, but finds better description in the MS(3) model.

Furthermore for a complete understanding of all the regimes estimated through what has been shown to be the better model specification it is useful to turn to Table 7, where the coefficients defined for every regime are shown.

In looking at the first two equations for the Japanese market (the exchange rate equation and the stock prices) it is possible to observe how in regime 2 the coefficients are generally higher in absolute value, suggesting a stronger influence of these 2 variables on each other. The signs are mostly similar, suggesting that changes in one variable are usually followed by changes of the other variable in

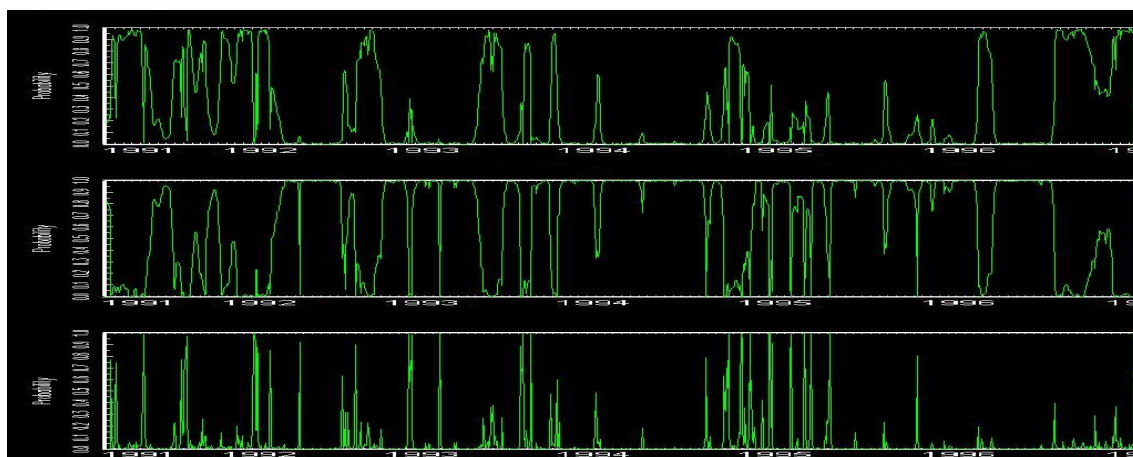


Figure 3: JAP-US: Detail of the smoothed probabilities

a coherent direction. So it is possible to deduct that one of the regimes is in some measure more unstable in its performance than the other, for shocks in variables are followed by shocks in the other or greater magnitude than in the first regime. Finally the third regime has on the other hand completely opposite signs, phenomena to be interpreted in communion with the fact that its duration is lower and it therefore most likely functions in the role of a “transition” regime.

Having described the empirical results of the alternating regimes, it is interesting to test for Granger causality in different periods. The question to answer is whether the same results found in the first part of this paper are confirmed for all the regimes or are typical of one or more regimes. In order to proceed towards this goal it is important to make a technical comment. The Figures shown in this paper may appear as if the regimes are not stable whatsoever, on a first look. This is only due to number of observations used to test the variables. The Japanese market has been studied over 4000 daily periods, and therefore a whole overview of all the data will overlook the details. This is why it is preferable in this case to work on Figure 3 an enlargement of a small part of Figure 2.

It is possible to notice how definite periods in which a regime prevails over the other are now shown. The number of observations in these separate periods is enough to allow a Granger causality test of the same nature as in the previous sub-section.

The sections taken in analysis in this case are the periods 15 through 70 (end of 1991 through February 1992) and 1300 through 1400 (October 1996 through

Regimes	Sub-periods	Dependent variables		
		S	A	$A^{US}$
Regime 1	21 Nov. 1991 - 6 Feb. 1992	0.5498	0.8896	0.2819
	24 Oct. 1996 - 13 Mar. 1997	0.4894	0.3856	0.9463
Regime 2	14 Apr. - 22 Dec. 1994	0.0557	0.0000	0.3256

Table 8: JAP-US market: causality test on different subperiods. Probabilities that  $H_0$  is true.  $H_0$ : dependent variable not caused by the others

March 1997) for regime 1 and between 640 and 820 (April-December 1994) for regime 2. Indeed in these samples the probabilities of being in the regimes studied are close to 1 throughout the single samples. As far as the samples for regime 1 the multivariate Granger test shows no causality in any direction, so that the causality seen in the linear model must not be characteristic of regime 1. In both the samples the data is strongly accepting the non causing hypothesis of the dependent variables in all the equations. This means that if the causality direction is to be found at all in this data (which could be expected, given the results of the test on the linear VAR) it is to be found in regime 2. Indeed regime 3 does not show long enough periods to be analyzed in a solid Granger test, therefore it could be hypothesized that any causality relationship will not be originated by this lower-persistent regime.

Furthermore as expected regime 2 shows a multivariate causality that goes from the variables involved in the stock market equation to the stock returns, confirming the results obtained in the linear analysis. However there is an unexpected trend found: the hypothesis that the exchange rate variable is not caused by the two other series cannot be rejected at the 6% level of significance. This result is however on the border-line and cannot be interpreted in any definite manner as it is.

### 3.3.2 The British market

An analogous study can be made for the British market, that has a parallel scenario. This market has been analyzed with similar models, a MS(2)-VAR(3) and a MS(3)-VAR(3) process, and the results are shown in the Equations 9 and 10. Precisely, the following matrixes show the results given by the Markov analysis

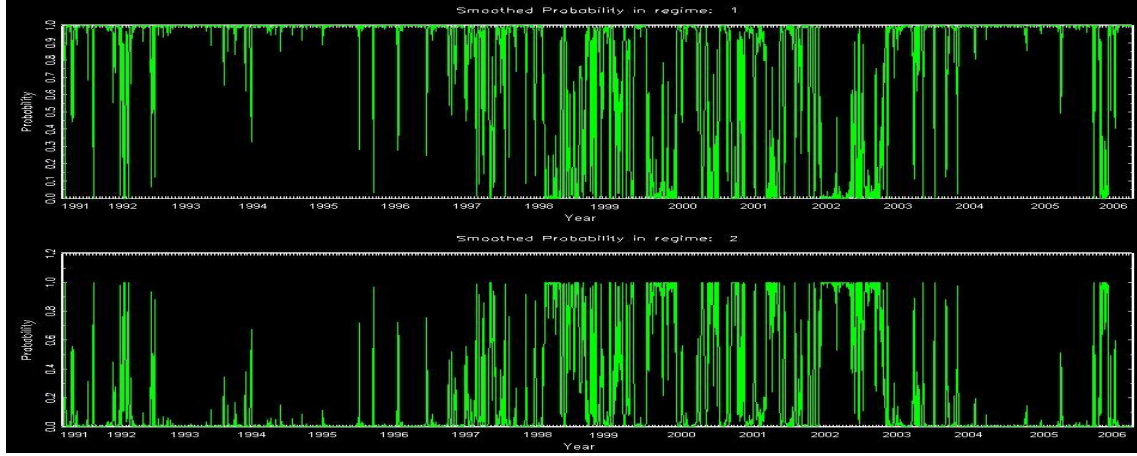


Figure 4: UK-US: Smoothed probabilities of being in the two regimes defined at every period in time

operated with 2 time-varying regimes.

$$P^{UK}[i, j] = \begin{bmatrix} 0.964 & 0.141 \\ 0.036 & 0.859 \end{bmatrix} \quad \xi \quad P^{UK} = \begin{bmatrix} 0.796 \\ 0.204 \end{bmatrix} \quad (9)$$

From this first set of information it is possible to remark that if there were to be two regimes, one would be a very strong and persistent one, while the second one would be less persistent and would only appear in about 1/5 of the periods. This situation is illustrated graphically in Figure 4. However the procedure adopted for choosing the number of regimes is based on the relationship between the performances of more explanations of MSM. Therefore the next set of information is needed to make a decision.

$$P^{UK}[i, j] = \begin{bmatrix} 0.371 & 0.075 & 0.007 \\ 0.616 & 0.913 & 0.042 \\ 0.012 & 0.012 & 0.951 \end{bmatrix} \quad \xi \quad P^{UK} = \begin{bmatrix} 0.087 \\ 0.713 \\ 0.200 \end{bmatrix} \quad (10)$$

Figure 5 describes these matrixes with a per-period illustration. In the 3 regime scenario it is interesting to notice a peculiar situation: the first regime appears to be not persistent at all, since it repeats itself only barely more than 1 in 3 times. However most of the times it yields to the second regime. This regime, in turn, in the case when it does not repeat itself, is very much more likely (over 6 times more) to switch back to the first regime, rather than to yield to the third regime.

In addition the probability that both regimes yield to the third regime is exactly equal. From this data alone it is intuitive that the first and second regimes might have a good chance of being an over-explication of one single regime, but nothing can be concluded without carefully observing the data from the two regimes.

Going back to Equation 9 it is indeed important to notice that the sum of the final probabilities of the first and second regime (in the 3-regime context) appear to be strikingly close to the unconditional probability of the first regime (in the 2-regime context). As a result, the unconditional probability of the remaining regime in both the systems is noticeably close as well. As a definite last test of this phenomenon it is useful to compare the two figures mentioned above (Figure 4 and 5). Indeed the last regime appears to be realized in the same periods in both contexts, leaving no doubt that an inclusion of a third regime in this seemingly two regime system would only confuse the picture, and upset the specification of the two alternating VAR specifications<sup>5</sup>. As a matter of fact, once the system is analyzed with two regimes, the first regime augments its persistency by far, allowing very good ground for Granger testing the series<sup>6</sup>.

However, since the choice of regimes has been made, it is first of all interesting to view the exact specification of the VAR coefficients. This operation will help characterize the two regimes in their differences as well as their similarities.

Table 9 shows the chosen specification for the British market. It appears that the two regimes are indeed very different one from another, and the one that is less persistent and less present in the time series (regime 2) has an economic reason for being so. Looking at the coefficients it is shown that regime 2 has in the vast majority of the cases higher absolute values. Given the nature of the model, which studies the reactions that the changes in variables have to the changes in other variables, higher coefficients mean higher instability in terms of changes. Bigger changes among the variables. It is no surprise then, that the same regime that may exhibit an array of fairly high coefficients of change between the variables, is also the one that often subdues to the more stable regime, with lower values and consequently with minor reactions to shocks in the system.

Furthermore, in a point of view of direction of change, the two regimes are

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<sup>5</sup>Indeed this phenomenon appears with just the same characteristics in an artificially created setting, that is being analyzed with more than the real underlying number of regimes.

<sup>6</sup>As a peculiarity to add to this choice of model, when the Granger analysis is tried on the 3 regime scenario the dates appear more confused, and the samples chosen are different. This causes different results which add no value over the linear solution. It will be shown later that on the other hand this models allows the researcher to detect new Granger causality directions just by studying the two regimes.



	S	A	$A^{US}$
C (Reg.1)	0.0023	0.0466	0.0621
(Reg.2)	-0.0009	-0.0359	-0.0512
S(-1) (Reg.1)	0.0143	0.0136	0.0511
(Reg.2)	0.1474	-0.1104	-0.1285
S(-2) (Reg.1)	-0.0119	-0.0204	0.0182
(Reg.2)	0.0209	-0.4082	-0.0012
S(-3) (Reg.1)	0.0018	0.2523	0.0080
(Reg.2)	-0.0348	0.5127	-0.0339
A(-1) (Reg.1)	0.0048	-0.0490	-0.0495
(Reg.2)	0.0078	-0.0476	0.1008
A(-2) (Reg.1)	-0.0099	0.0087	0.0019
(Reg.2)	-0.0108	-0.2897	-0.0116
A(-3) (Reg.1)	0.0154	-0.0024	-0.0355
(Reg.2)	0.0241	0.2281	-0.0213
$A^{US}(-1)$ (Reg.1)	-0.0515	-0.0138	0.0361
(Reg.2)	0.0908	-0.2031	-0.0186
$A^{US}(-2)$ (Reg.1)	-0.0057	0.0066	-0.0090
(Reg.2)	-0.0039	-0.4358	-0.0462
$A^{US}(-3)$ (Reg.1)	0.0082	0.0011	-0.0371
(Reg.2)	0.0007	0.2401	-0.0054

Table 9: UK-US market: MS(3)-VAR(3) specification

Regimes	Sub-periods	Dependent variables		
		S	A	$A^{US}$
Regime 1	18 Mar. 1993 - 31 Aug. 1995	0.4443	0.0000	0.8934
	10 June 2004 - 5 Jan. 2006	0.6149	0.0000	0.5473
Regime 2	2 Jul. - 24 Sep. 1998	0.0355	0.0238	0.0580
	13 Jun. - 31 Oct. 2002	0.4020	0.0000	0.2966

Table 10: UK-US market: causality test on different subperiods. Probabilities that  $H_0$  is true.  $H_0$ : dependent variable not caused by the others

also different. Although not in a majority of the cases, it is possible to see that in about half the coefficients the direction of change provoked by changes in other variables is opposite depending on the regime that is realized at the period. This is very important from an economic point of view because it depicts one of the main reasons why MSM became so popular in describing time series from financial markets. By being able to catch the different moods of the market these models understand when the market behaves in different ways given similar shocks.

Having described the chosen model it is interesting to proceed the study of this market into reviewing Granger causality and comparing it to the results obtained through the linear estimation applied earlier in this paper. According to Figure 4 the samples chosen will be from 360 through 1000 (March 1993 - August 1995) and 3290 through 3700 (June 2004 through the beginning of 2006) for regime 1 and 1740 through 1800 (July through September 1998) and 2770 through 2870 (June through October 2002) for the testing of the second regime.

Table 10 shows the results for this market. In the first regime the data appears to confirm completely the results obtained from the linear analysis. The local stock market is caused by the other two variables in both samples analyzed. However it is interesting to notice that in the second regime the two samples have slightly different results. Indeed while the stock market is always caused by the other two variables, in the first sample, that goes from June to September 1998 the exchange rate is definitely caused by the other two variables as well. This is very important since it allows the reader to detect a new dynamics that appears in the data. Although this result is not confirmed in the second sample, it pinpoints a new phenomenon within these data<sup>7</sup>.

<sup>7</sup>In a nutshell it is possible to notice how for the first time the probability of the American

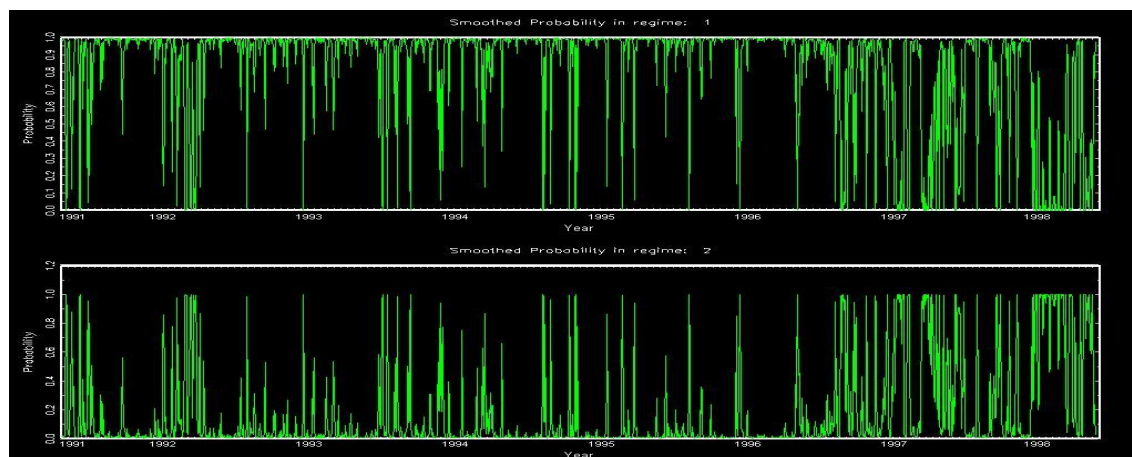


Figure 5: GER-US: Smoothed probabilities of being in the two regimes defined at every period in time

### 3.3.3 The German market

The last market to be analyzed (the German-US) has been studied over a shorter range than the others<sup>8</sup>. The methodology applied will be similar than in the others. Light can be shed on the data in study from the first set of information, listed in Equation 11.

$$P^{GER}[i, j] = \begin{bmatrix} 0.9312 & 0.3242 \\ 0.0688 & 0.6758 \end{bmatrix} \quad \xi \quad P^{GER} = \begin{bmatrix} 0.8248 \\ 0.1752 \end{bmatrix} \quad (11)$$

It appears from a first look that there will be a very strong regime, that appears over 4 times out of 5 in the system, while a weaker, less persistent regime complements it, in some periods. Figure 5 will show in detail the situation described by these matrixes. As the analytical situation shows (from the low persistency of the second regime), it is hard to pinpoint longer periods in which a regime takes a lead over the other. However in order to make a decision on the model to choose the next set of information is needed where the three regime scenario is studied.

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stock being caused by the other two variables is higher. However from this data is not possible to conclude that the phenomenon is statistically relevant, since the hypothesis of non-causality cannot be rejected at the 5% level of significance.

<sup>8</sup>In a nutshell this has the positive effect of making the graphs clearer and the long persistent periods easier to find.

This is listed in Equation 12.

$$P^{GER}[i, j] = \begin{bmatrix} 0.980 & 0.014 & 0.014 \\ 0.011 & 0.904 & 0.283 \\ 0.010 & 0.082 & 0.703 \end{bmatrix} \quad \xi \quad P^{GER} = \begin{bmatrix} 0.4100 \\ 0.4520 \\ 0.1379 \end{bmatrix} \quad (12)$$

It is very interesting to notice how there appears to be a fairly strong cycle between the second and third regime. This could be a hint that maybe these regimes would be better grouped as one, but all data needs to be first objectively checked. For example the persistency of all the regimes in this system is higher (even for the least persistent one) than in the two regime explanation of the data. So it is important to verify how the final probabilities add together to check whether the hypothesis of grouping the second and third regime into one could be verified without further concerns.

However from the matrixes it is clear that not only the unconditional probabilities do not add up to the same result as in the two regime scenario, but also something more would be missed. The first of the two regimes in the first set of info is indeed much less persistent than it would be in the second model. Furthermore the two last regimes from the second model are a lot more persistent than the sum of them would be together if they were one as shown in the first model. It is crucial to remember that once the number of regimes is chosen (and the VAR structure is singled out among other statistical models) there is a univocal correspondence with the definition of the model. Explicitly, the coefficients chosen for the models are selected given the information of the data. So grouping the two regimes together does not give the results obtained. Still, an explanation is needed to understand exactly what is going on in the data and why the cycle between regime 2 and 3 appears so clearly.

This explanation is given by Figure 6, which shows the detailed period per period subdivision of the time series in regimes. The reason for the strong cycle is simply that in the second half of the time series the first regime disappears completely, letting the system alternate between the remaining two regimes only. Given that the probabilities are only a a-posteriori count of the times when the two regimes switched between each other, the question is answered<sup>9</sup>. So, given

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<sup>9</sup>As a counter prove of what has been stated here it is useful to look at the situation from a different point of view. In an hypothetical two regime hidden system the second regime would take the complete lead in the second part of the series, while the first would be leading the first part. This does not in fact happen in the two regime scenario because the lack of the third regime changes the coefficients and makes them more general, so that the big break of period 1033 cannot even be noticed in the graph.

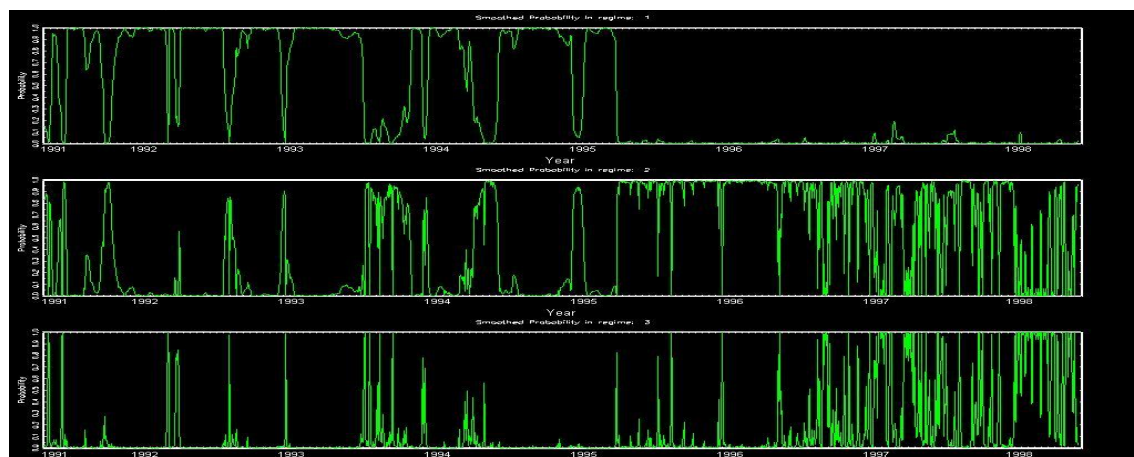


Figure 6: GER-US: Smoothed probabilities of being in the three regimes defined at every period in time

the data analyzed and the considerations originating from the data, the German market will be analyzed with a MS(3)-VAR(2) model.

Looking in depth at this model through a further study of Equation 12 it is useful to add some information that will help in the interpretation of the market. To be highlighted is a very stable regime (number 1) for its high probability to repeat itself. Given this peculiar regime it is interesting to notice how these probabilities are calculated. Since they are calculated after the regimes are already set in the Expectation Maximization cycle to determine the regimes and their specification, these are a simple count of all the times that the regime is in place over all the times that it is not. Therefore the correct meaning of the 98% shown below is that in exactly that percentage of times that regime 1 is the status of nature, the next status in nature is also going to be regime 1. This is a clue concept as far as predicting the system, especially if teamed with other considerations on this data.

The matrix indeed also shows other trends: regime 3, the one that has the lowest of all the conditional probabilities could maybe be referred to as the “transition regime”. The reason for this is that it does not have long periods in which it is leading over the other regimes, but its importance has been proven above as the one regime without which the system would not be well defined. Due to this regime, the other two can be defined so as to have high persistence levels, and lead the system very often. This third regime could, however, take an important role in the description and interpretation of the time series operated in the next section.

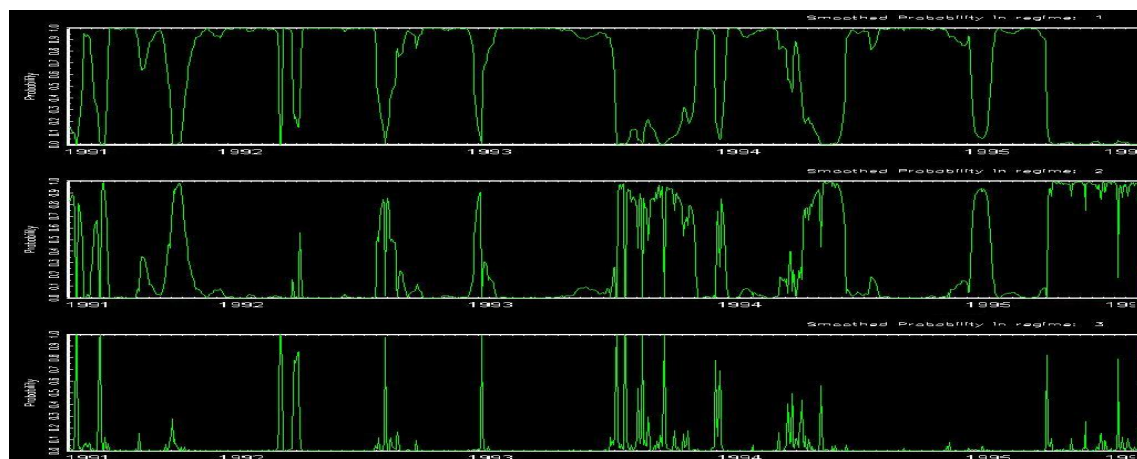


Figure 7: GER-US: Detail of the smoothed probabilities

This concept can be verified by looking at the overall picture of probabilities over time (Figure 6). Regime 1 does not have many periods in which it has probabilities close to 1; however, whenever it is so, those periods are long and continuous. This phenomenon is clear in Figure 7, an enlargement of the previous graph, where the continuous periods from regime 1 are shown in detail. As the graphs show, this is positive for a numerical study of the data in that regime.

In taking advantage of this situation, the application of the Granger test in this market for this regime may be done over several continuous periods of time. These will be: 121-225 (April-September 1992), 250-330 (October 1992- February 1993), 445-580 (July 1993- January 1994) and 810-945 (December 1994 through June 1995) as far as the tests on regime 1. On the other hand in analyzing regime 2 it is possible to take one larger period that is only slightly broken by the brief interferences of regime 3. This will be the period between 1033 and 1321 (October 1995- November 1996). Finally, in this market it is also possible to show causality directions for the third regime, which will be studied in the sample from period 1750 to 1800 (July to September 1998).

As shown in Table 11, while regime 2 maintains the unidirectional causality of the exchange rate and American returns causing the local returns on the stocks, regime 1 has mixed results on almost all the samples. In particular the hypothesis of non-causality is rejected clearly in only one case, and in another is not accepted if considering a confidence value below 99%. The other two cases show a scenario in which  $H_0$  is accepted and one in which it is only accepted for values below 94%.

Regimes	Sub-periods	Dependent variables		
		S	A	$A^{US}$
Regime 1	17 Apr. - 10 Sep. 1992	0.8926	0.0838	0.9312
	15 Oct. 1992 - 4 Feb. 1993	0.8245	0.0153	0.8434
	15 Jul. 1993 - 20 Jan. 1994	0.8426	0.1153	0.2005
	8 Dec. 1994 - 15 Jun. 1995	0.1127	0.0000	0.0825
Regime 2	17 Oct. 1995 - 22 Nov. 1996	0.6247	0.0000	0.6243
Regime 3	16 Jul. - 24 Sep. 1998	0.0310	0.0000	0.7886

Table 11: GER-US market: causality test on different subperiods. Probabilities that  $H_0$  is true.  $H_0$ : dependent variable not caused by the others

On the other hand the sample from regime 3 shows a 2-way causality, from the US stock prices and the exchange rate to the local stock market as well as from the stock market prices to the exchange rate. The fact that regime 3 is only leading the system in a minority of the cases shows the reason why the test on the whole sample operated in the first part of this paper did not catch the second causality direction.

A look at Table 12 shows that on average once again regime 2, the one where the causality direction from  $S$  and from  $A^{US}$  towards  $A$  is more certain, has higher coefficients than regime 1. At the same time regime 3 has on average more extreme coefficients than the other two. However in this market these comments may be made with a smaller margin in the magnitudes compared to the Japanese market, for example. This consideration could provide more ground for future research. On the other hand in this market there is a strong difference in the signs of the two more persistent regimes in both the equations of interest (the local stock market and the exchange rates). This is a basic difference that corroborates the fundamental, radical diversity in nature of these two regimes, as shown by the two figures. The German market is indeed a clear example of how a system can alternate between different regimes that are characterized in diverse manners without these being completely opposite.

## 4 Interpreting the results

In the previous part of the paper the markets have been studied through different models, the best analysis for every market has been chosen, and other empirical

	S	A	$A^{US}$
C (Reg.1)	0.0179	-0.0293	0.0536
(Reg.2)	-0.0452	0.1897	0.1319
(Reg.3)	0.0667	-0.2511	-0.1707
S(-1) (Reg.1)	-0.0189	-0.1853	-0.0132
(Reg.2)	-0.0811	-0.2047	-0.0204
(Reg.3)	0.2476	-0.5577	-0.0368
S(-2) (Reg.1)	-0.0687	0.0303	0.0203
(Reg.2)	0.0322	-0.1627	-0.0001
(Reg.3)	0.0831	-0.1901	-0.0092
A(-1) (Reg.1)	0.0311	0.4539	0.0005
(Reg.2)	-0.0316	0.4917	0.0630
(Reg.3)	-0.1051	0.8576	-0.1060
A(-2) (Reg.1)	-0.0134	-0.0472	0.0031
(Reg.2)	-0.0150	-0.0769	-0.0682
(Reg.3)	0.1710	-0.1826	-0.1432
$A^{US}(-1)$ (Reg.1)	0.0373	0.0958	-0.0452
(Reg.2)	0.0213	-0.0549	-0.0303
(Reg.3)	0.0245	-0.1138	-0.0224
$A^{US}(-2)$ (Reg.1)	0.0337	0.0040	-0.0425
(Reg.2)	-0.0236	-0.0944	-0.0135
(Reg.3)	0.0205	-0.0808	-0.0301

Table 12: GER-US market: MS(3)-VAR(2) specification



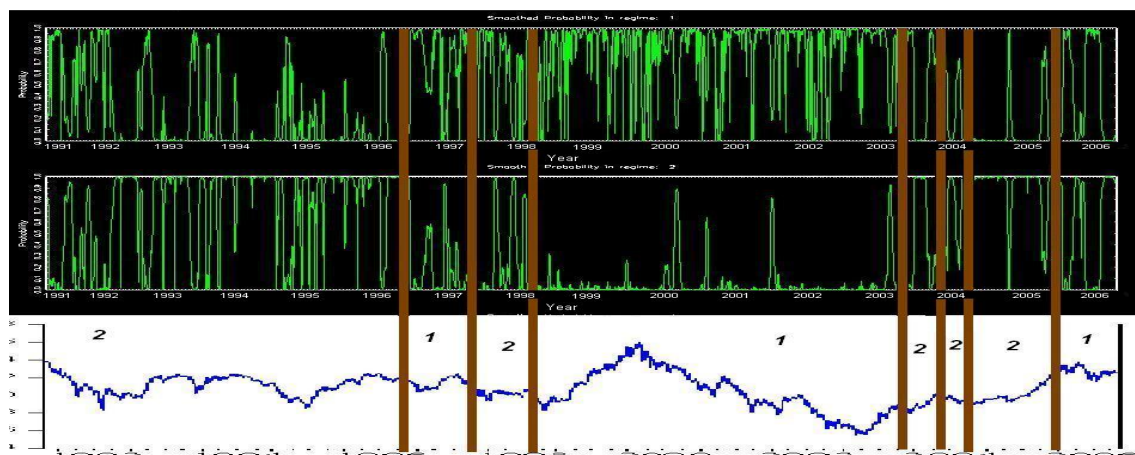


Figure 8: JAP-US: The three regime setting compared to the Japanese stock market prices

tests were applied on the data (Granger causality tests). This section takes a different approach. It starts from the selected models and it interprets the results found, comparing them with real events in the economy. First of all the markets will be observed in detail singularly, and a relationship will be traced, if possible, towards the underlying local stock market as a measure of the local economy. In the second part of this section an overall review of all the events will be looked at from a larger distance, including all the markets together. This will be interesting since all the case studied taken in consideration are indeed important players in the world economy.

#### 4.1 Single market observations

The Japanese analysis shows an interesting relationship when compared to the local stock market. The two main regimes (which are the ones that together account for over 90% of the time series) alternate in such a way that whenever the system is in regime 2 the local stock market seems to be in a calmer “mood” or status. For almost the complete first part of the series it is possible to see a strong lead of this stable regime, and the dynamics of the market is indeed minor, if looked at in daily observations over a longer period (years). This is suddenly upset whenever the status switches to the first regime, in occasion of the dissolution of the Parliament by Hashimoto (September 26th, 1996) and the consequent elections (October 20th, to be compared to the model switch on October 21st).

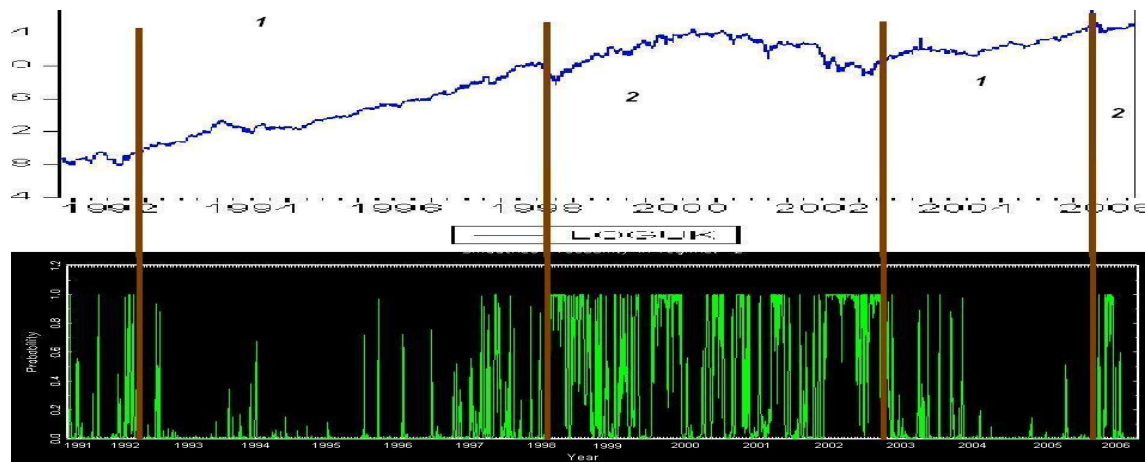


Figure 9: UK-US: The two regime setting compared to the British stock market prices

In the following period the regime remains leading as the Japanese Central bank tries to counter the statistically important fall in the value of the Yen [53]. This consideration is in harmony with what has been found through the observation of the coefficient magnitudes. Indeed the first regime has generally higher coefficients in absolute value than regime 2. Throughout the middle of the time series it is possible to see the large daily changes in the local stock market, which seems to be more sensitive to shocks whenever in this regime. Finally in the rest of the series, throughout close to the end, the dynamics switches back to a flow similar to the beginning, with regime 2 leading in a decisive way towards a calmer course. The leading of regime 1 in some instances has a lot of characteristics proper of a behavioral bubble, and could be originated by an excessive sensitivity of the market to signals given by the main market players.

On the other hand, the British scenario has a different interpretation for the regimes compared to the Japanese one. This is because the local market seems to be always in a fairly strong shape, with returns fairly high. Therefore when regime 1 is leading the market, the prices are rising at a moderate, but consistent pace. However when the system switches to the second regime this is no longer true. It appears that all signals, negative and positive, given from the market are magnified. So it appears that in the beginning the growth rate is steeper than in regime 1. Furthermore this steeper growth is followed by an actual set back of the prices. The result is that when the system switches back to the first regime the log-level of

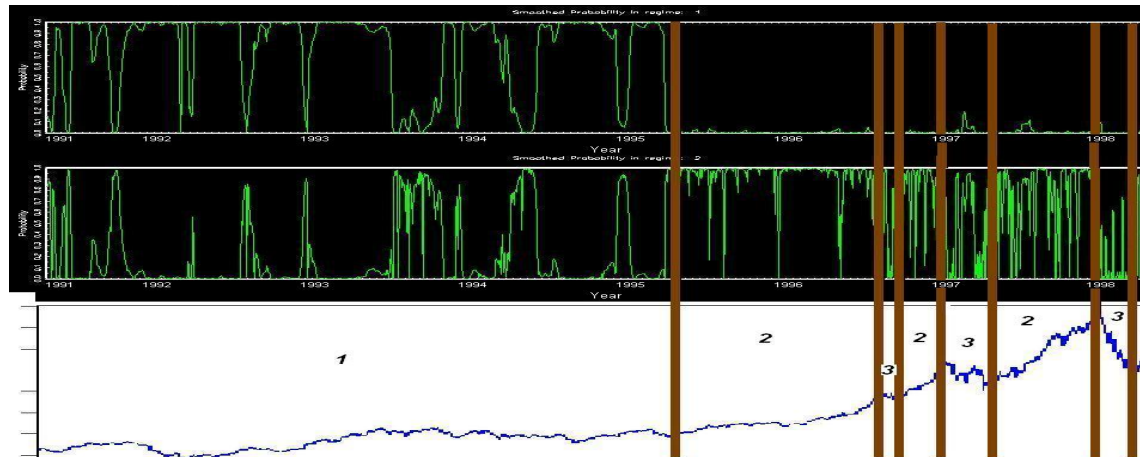


Figure 10: GER-US: The three regime setting compared to the German stock market prices

the stock prices is only slightly higher than when regime 2 first appeared. Finally it is indeed interesting to notice how the log-growth of the market is just about the same in the two instances of regime 1.

An important point to make about this market is the relationship that these switches have been having to the international scenario: it is reasonable to hypothesize that the switches in this market are bonded to the events in the European economy. It is a fact that the more unstable regime of the two took over just before and after the introduction of the Euro in Continental Europe. This might be a factor. In any case, this regime also corresponds to a complete cycle in the returns of the British stock market.

The last scenario to check against its local real economy is the German one, which has been observed to the beginning of 1999 just to avoid such a large statistical change as the introduction of the new currency being in the picture. This market yields very interesting results when the MS probabilities are compared to the local economy. Up to the end of 1995 (October) the first regime leads the scenario, and it is possible to see a very calm situation on the levels of the German stock prices. However after that date the other two regimes take over: the second one will always represent a growth regime, while the third regime will always be in correspondence of a fall of the prices. This is very interesting since the markets seem to be more subject to steeper growth as well as depression since the beginning of 1996. At the same time, what used to be a fairly stable regime, the first one,

disappeared as the German market got closer and closer to the switch to the Euro currency. This regime division would be a puzzle, if it was not for its strikingly clear relationship to the price levels of the German stock market: the economy underwent changes in its specification, and the previous status simply could not describe the pressures of the market well enough anymore. This property to be able to catch the changes of mood in a market is a very important one in the use of MSM<sup>10</sup>.

In order to fully understand the clear cut change of the model in October 1995 and the further switch in the end of 1996 it is interesting to look at the financial news that have been important to the market in that period [53]. The German economy entered September 1995 with a low inflation and rates just cut by the Bundersbank to the lowest value in over six years (August 24th, 1995) as an answer to “a weaker than expected money supply trend” (FT). Furthermore during the three months between September and November a series of negative news hit the market. First the market belief on the EMU is affected negatively by the statements of high officials to comment on a higher risk of stability if the countries involved do not improve their situation (Sept. 22nd and 25th, 1995). Then, during October and November, the German rate of unemployment (October 5th, 95, referring to September data) and of industrial production (November 6th, 95, referring to October data) raise several worries as data on eastern Germany looks worse than expected. This information shows the underlying reality of the financial markets (both exchange rate and stock market) during that period, and sheds light over the switch in the specification operated by the MSM in October 1995. It is also possible to remark that a surprise fall in industrial production (November 4th, 1996) and a consequent pessimism hitting the market [53] happen in corrispondence of the switch between the second and the third regime noticed in the end of 1996 and may therefore be looked at as likely causes for the model’s different specification.

## 4.2 An overall interpretation of the period

This study has analyzed in depth three markets that are important in a world point of view of financial time series. For this reason it is interesting to observe the similarities and differences among the regimes at certain points in time. This

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<sup>10</sup>From this interpretation it is possible to see the role of the third regime, which in this case appears to be not a “transition” regime, but takes active part in describing longer periods of the series starting in the second half of the series.

is just the goal of this last section before the conclusions.

Looking at the dates it is peculiar (but not so much after all if looking at the world financial interactions) how many of the trends are repeated in all the markets seen. The German and Japanese market, for example, hold the more stable regime throughout the first part of the series, up to 1996 (slightly earlier for Germany and a few months later in Japan). The British market also has the more stable regime leading the scene in the first part of the series, and this yields to the more unstable status only in 1998. This is also an important date since on the Japanese market the stable regime does return for another year, and then disappears in 1998 through the end of 2003. This is also the year when the more stable regime comes back in the UK market. It is not possible through this data to speculate when the calmer regime would come back in the German market simply because of the different sample chosen for this economy.

In conclusion, all the three markets show a more stable period throughout the first part of the 1990s and then have a more unstable regime. The situation will only reverse at the end of 2003, for the first two markets studied. These similarities between the different economies are very positive for two complementary reasons. On the one hand they show the intrinsic connection between advances financial markets worldwide, while on the other they show that the great flexibility of the models applied does indeed catch real changes in the market and helps the researcher break the analysis in such a manner that the real underlying economy is well described in a consistent manner throughout the different scenarios.

## 5 Questions for the future and conclusions

This study has analyzed three different cases in the characterization of the relationship between stock returns and exchange rate returns. The analysis started from a linear VAR process and led to a non-linear, Markov Switching process. Throughout the paper the concept of direction of causality has been central, and it has been applied to both the linear VAR and the non-linear model. Finally an overall interpretation of the results was provided with a comparison to the local stock prices. The findings of the study have been of multiple nature. On one hand the non-linear approach has indeed helped obtain more information from the data. The data has been approached in the second half of the paper by a single model that could describe it in an accurate manner, pinpointing in certain cases where the Granger causality was present and where it was not (or in which direction it was, within the time series). The different segments of data were described and

analyzed to seize common traits. At the same time, just as the non-linear analysis gave more information on the data, so the data has characterized the non-linear models in certain ways that may provide field for future research.

Some of the questions that arose throughout the paper have been the following. About a methodology to discern the Markov Switching model to be used. The number of regimes is a very important variable in this models. However only limited research has been carried out to choose the number of regimes applicable case by case. In most other situations an older test has been chosen that traditionally helps choose between two general models and a way has been researched to adapt that test. However the MSM nature is different than other models, and needs a procedure typical of this logic, and not a general model selection procedure, to pinpoint exactly the number of regimes to be used. The method used in this paper takes certainly the information given from the MS analysis, and uses that information to make an objective decision given those results. However there is no in this procedure, a specific statistic that will select a number of regimes over all the others. It is still a comparison between two different MS models (like the Likelihood ratio test would be, although it has the advantage of using the typical information provided by this specific analysis). Ideally a new statistic could be developed that in the spirit and logical pattern of MSM would single out the best number of regimes (given the theoretical model chosen) among a wider selection than just two per time.

Secondly, concerning the presence of the third, “transition regime”: can it be characterized further than just transitional regime? Is there an economic reason for which when it is present it has a lower persistency? Is it just subject to this single markets analyzed or is it a concept that may be generalized? In this paper two of the three markets have been analyzed by a 3-regime MSM. However in the German market the third regime had the very precise purpose to describe the decreasing prices in the local economy, which could hardly be described as “transitional”. On the other hand the Japanese market, although was not well defined by a 2 regime specification, had the third regime take the role of a smoothing status between the other two, more persistent regimes. Therefore from the data and analysis performed in the paper it appears clear that systems may require such a third regime, and that the model would not be well specified without, however an economic meaning for the third regime (when this is not clear as in the German case) is yet to be researched. Also several other examples in literature on MSM often refer to a 3 regime system rather than 2 or 4 regimes. However only 2 of them generally have high persistence in the alternating of the regimes. This shows that

this question is not relevant only to this particular field, but can be of importance in a wider array of applications.

Thirdly, in the description of the local stock returns does exchange rate and the overall stock market unidirectional causality create more volatility? This could be explained by the fact that the causality direction does not explain the everyday fluctuations of the stock market, but adds more on top of the ones that the stock market is already subject to. In other words there are many sources to market volatility, and the multivariate causing detected in the paper is but one of such sources. This is beneficial to the purpose of this paper since the main objective is to analyze if more understanding of the stock returns (as well as the exchange rate market) can be achieved through the contemporaneous study of both markets. In that case according to the data studied in this paper one of the sources of change of the stock market is the causality uncovered. However, as the previous section shows, this source of change is not present at all times, and when it is present its effect is most likely added on the other sources of variation.

In conclusion, the study has shown the alternating over time of regimes in the relationship between exchange rates, local stock returns, and American stock returns, taken as a proxy for world (exogenous) stock market. The analysis carried out has allowed the determination and specification of the regimes in terms of variable coefficients as well as change over time. Indeed, while it is possible and reasonable that the relationship studied follows a time-varying, non-linear structure, few are yet the empirical papers based on MSM procedures, which to this day stand in the literature as examples of flexible, versatile models.

## References

- [1] Issam S.A. Abdalla and Victor Murinde. Exchange rate and stock price interactions in emerging financial markets: evidence on india, korea, pakistan and the philippines. *Applied financial economics*, 7:25–35, 1997.
- [2] R. Aggarwal. Exchange rates and stock prices: a study of the us capital markets under floating exchange rates. *Akron Business and Economic Review*, Fall:7–12, 1981.
- [3] Carlo Altavilla and Paul De Grauwe. Non-linearities in the relation between the exchange rate and its fundamentals. *CESifo Working Papers*, Category 6:No. 1561, 2005.
- [4] Patricia Alvarez-Plata and Mechthild Schrooten. The argentinean currency crisis: A markov-switching model estimation. *German Institute for economic research*, Berlin, Discussion paper 348, May 2003.
- [5] Torben G. Andersen, Tim Bollerslev, Francis X. Diebold, and Clara Vega. Real-time price discovery in stock, bond, and foreign exchange markets. *National Bureau of economic research*, May 2005.
- [6] K. N. Badhani. Dynamic relationship among stock prices, exchange rate, and net fii: Investment flow in india. 2005.
- [7] Mohsen Bahmani-Oskooee and Ahmad Sohrabian. Stock prices and the effective exchange rate of the dollar. *Applied economics*, 24:459–464, 1992.
- [8] Bhattacharya Basabi and Mukherjee Jaydeep. Causal relationship between stock market and exchange rate, foreign exchange reserves and value of trade balance: A case study for india. 2001.
- [9] Benoit Bellone. Classical estimation of multivariate markov-switching models using msvarlib. July 2005.
- [10] J. Bilson. The monetary approach to the exchange rate - some empirical evidence. *IMF Staff Papers*, 25:48–75, 1978.
- [11] W.H. Branson. Macroeconomic determinants of real exchange rate risk. *Herring, R.J., (Ed.), Managing Foreign Exchange Rate Risk. Cambridge University Press, Cambridge, MA*, 1983.



- [12] Yin-Wong Cheung and Ulf G. Erlandsson. Exchange rates and markov switching dynamics. *CESifo Working paper No. 1348*, Category 6: Monetary policy and international finance, December 2004.
- [13] Brad Comincioli. The stock market as a leading indicator: An application of granger causality. *The University Avenue Undergraduate Journal of Economics*, 1996.
- [14] Hans Dewachter. Can markov switching models replicate chartist profits in the foreign exchange markets? *Journal of International Money and finance*, 20:25–41, 2001.
- [15] Desislava Dimitrova. The relationship between exchange rates and stock prices: Studied in a multivariate model. *Issues in Political Economy*, 14, August 2005.
- [16] Rudiger Dornbusch and Stanley Fischer. Exchange rates and the current account. *The American Economic Review*, 70 (5):960–971, December 1980.
- [17] C. Engel and J. D. Hamilton. Long swings in the dollar: are they in the data and do the markets know it? *American Economic Review*, 80:689–713, 1990.
- [18] P. Franck and A. Young. Stock price reaction of multinational firms to exchange realignments. *Financial Management*, 1:66–73, 1972.
- [19] J.A. Frankel. Monetary and portfolio balance models of exchange rate determination. In: *Bhandari, J.S., Putnam, B.H. (Eds.), Economic interdependence and flexible exchange rates. MIT Press, Cambridge, MA.*, 1983.
- [20] Clive W.J. Granger, Bwo-Nung Huang, and Chin Wei Yang. A bivariate causality between stock prices and exchange rates: Evidence from recent asia flu. *Discussion paper*, April 1998.
- [21] J. Hamilton. A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica*, 57(2):357–384, 1989.
- [22] James D. Hamilton. *Time Series Analysis*. Princeton University Press, 1994.
- [23] Bruce Hansen. Inference when a nuisance parameter is not identified under the null hypothesis. *University of Rochester Discussion Paper 296*, 1991.

- [24] Bruce Hansen. The likelihood ratio test aunder non-standard conditions: testing the markov switching model of gnp. *Journal of Applied econometrics*, 7:61–82, 1992.
- [25] Bruce Hansen. Erratum: The likelihood ratio test aunder non-standard conditions: testing the markov switching model of gnp. *Journal of Applied econometrics*, 11:195–198, 1996.
- [26] Abdulnasser Hatemi-Ja and Manuchehr Irandoust. On the causality between exchange rates and stock prices: A note. *Blackwell Publishers, Bulletin of economic research*, 54:2, 2002.
- [27] Abdulnasser Hatemi-Ja and Eduardo Roca. Exchange rates and stock prices interaction during good and bad times: evidence from the asean4 countries. *Applied Financial Economics*, 15:539–546, 2005.
- [28] Harald Hau and Helene Rey. Exchange rates, equity prices and capital flows. *Oxford University Press*, 2005.
- [29] Fumio Hayashi. *Econometrics*. Princeton University Press, 2000.
- [30] Soren Johansen. Estimation and hypothesis testing of cointegration vectors in gaussian vector autoregressive models. *Econometrica*, 59, No. 6:1551–1580, November 1991.
- [31] C.E. Jorion. The exchange rate exposure of us multinationals. *Journal of Business*, 63 (3), 1990.
- [32] Saadet Kasman. The relationship between exchange rates and stock prices: A causality analysis. 2003.
- [33] Hans-Martin Krolzig. Statistical analysis of cointegrated var processes with markovian regime shifts. *Institute of Economics and Statistics and Nuffield College, Oxford*, SFB373 Discussion Paper:25, 1996.
- [34] Hans-Martin Krolzig. *Markov-Switching Vector Autoregressions. Modelling, Statistical Inference, and Application to Business Cycle analysis*. Springer Verlag, 1997.
- [35] Allan P. Layton and Daniel Smith. A further note on the three phases of the us business cycle. *Applied Economics*, 32.9, July 2000.

- [36] J. P. LeSage. Econometrics toolbox. Matlab software.
- [37] Mark and Nelson. *International Macroeconomics and finance*. Blackwell.
- [38] Naeem Muhammad and Abdul Rasheed. Stock prices and exchange rates: Are they related? evidence from south asian countries. 2001.
- [39] Jesus Munoz and P. Nicholas Snowden. Floating without flotations- the exchange rate and the mexican stock market: 1995-2001. *Journal of International Development*, 18:299–318, 2006.
- [40] Carol L. Osler. Currency orders and exchange rate dynamics: An explanation for the predictive success of technical analysis. *The Journal of Finance*, October 2003.
- [41] Edoardo Otranto and Giampiero M. Gallo. A nonparametric bayesian approach to detect the number of regimes in markov switching models. *Econometric Reviews*, 21, No. 4:477–496, 2002.
- [42] Anna Pavlova and Roberto Rigobon. Asset prices and exchange rates. June 2003.
- [43] Kate Phylaktis and Fabiola Ravazzolo. Stock prices and exchange rate dynamics. 2002.
- [44] Richard Priestley and Bernt Arne degaard. Long swings in the dollar and the exchange rate exposure of stock returns. March 2006.
- [45] Zacharias Psaradakis and Nicola Spagnolo. On the determination of markov-switching regimes in markov switching autoregressive models. *Journal of Time series Analysis*, 24, N.2, 2003.
- [46] Bala Ramasamy and C.H. Matthew Yeung. The causality between stock returns and exchange rates: revisited. *Blackwell Publishing Ltd/University of Adelaide and Flinders University*, 2005.
- [47] Christopher A. Sims and Tao Zha. Mcmc method for markov mixture simultaneous-equation models: a note. 2004.
- [48] Christopher A. Sims and Tao Zha. *Were there regime switches in US monetary policy?* 2006.

- [49] C.E. Smith. Equities and the uk exchange rate. *Applied economics*, 24:327–335, 1992.
- [50] C.E. Smith. Stock market and the exchange rate: A multi-country approach. *Journal of Macroeconomics*, 14:607–629, 1992.
- [51] L.A. Soenen and E.S. Hennigar. An analysis of exchange rates and stock prices - the us experience between 1980 and 1986. *Akron Business and Economic Review*, Winter:7–16, 1988.
- [52] Daniel Stavrek. Linkages between stock prices and exchange rates in the eu and the united states. *Project No. GAR 402/02/1408*, 2004.
- [53] Massimo Tivegna and Grazia Chiofi. *News e dinamica dei tassi di cambio*. Il Mulino, 2000.
- [54] Sheng-Yung Yang and Shuh-Chyi Doong. The dynamic relationship between the stock prices and the exchange rates and the spillovers effect. 2003.
- [55] Sheng-Yung Yang and Shuh-Chyi Doong. Price and volatility spillovers between stock prices and exchange rates: Empirical evidence from the g-7 countries. *International Journal of Business and Economics*, 3 (2):139–153, 2004.
- [56] Wu Ying. Exchange rates, stock prices, and money markets: evidence from singapore. *Journal of Asian Economics*, 12:445–458, 2001.